

**Exercise 16**

Given  $f(x) = \frac{x}{x+2}$  and  $g(x) = \frac{2x}{1-x}$ :

- (a) Find  $f(g(x))$  and  $g(f(x))$ .
- (b) What does the answer tell us about the relationship between  $f(x)$  and  $g(x)$ ?
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**Solution**

To evaluate  $f(g(x))$ , plug the formula for  $g(x)$  where  $x$  is in the formula for  $f(x)$ .

$$\begin{aligned} f(g(x)) &= \frac{\frac{2x}{1-x}}{\frac{2x}{1-x} + 2} \\ &= \frac{\frac{2x}{1-x}}{\frac{2x}{1-x} + 2} \times \frac{1-x}{1-x} \\ &= \frac{2x}{2x + 2(1-x)} \\ &= \frac{2x}{2x + 2 - 2x} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

To evaluate  $g(f(x))$ , plug the formula for  $f(x)$  where  $x$  is in the formula for  $g(x)$ .

$$\begin{aligned} g(f(x)) &= \frac{2 \frac{x}{x+2}}{1 - \frac{x}{x+2}} \\ &= \frac{2 \frac{x}{x+2}}{1 - \frac{x}{x+2}} \times \frac{x+2}{x+2} \\ &= \frac{2x}{1(x+2) - x} \\ &= \frac{2x}{x + 2 - x} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

Since  $f(g(x)) = x$  and  $g(f(x)) = x$ ,  $g(x)$  is the inverse function of  $f(x)$ .